04th December

Completed Exercises from the lecture on

<Circles>

1. Easy, Page 2;

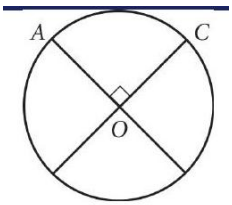
2. Medium, Pages 3-5;

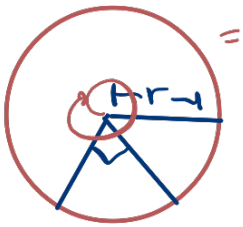
Can be found below. I solved some extra exercises to account for the lack of exercises.

Circles

Easy

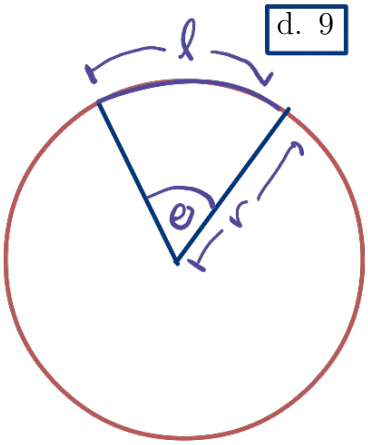
(1) **23c5fcce** MULTIPLE CHOICE One answer only

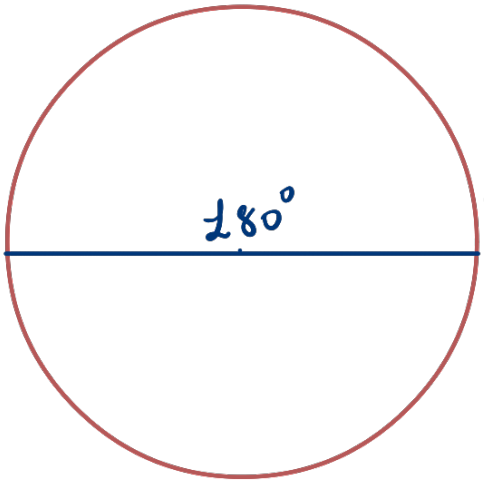




The circle above with center O has a circumference of 36. What is the length of minor arc AC ?

a. 18
b. 12
c. 36
d. 9





Medium

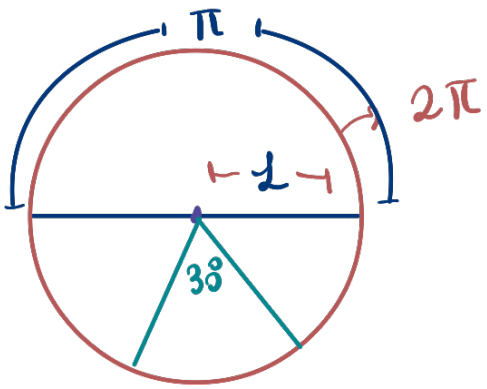
(1) **8e7689e0** SHORT ANSWER Case-Insensitive

The number of radians in a 720-degree angle can be written as $a\pi$, where a is a constant. What is the value of a ?

$180^\circ := \pi$ radians

$720^\circ = 4 \cdot 180^\circ = 4 \cdot \pi$ rad $= a\pi$ rad

$a = 4$



$180^\circ = \pi$ radians

(3) **856372ca** MULTIPLE CHOICE One answer only

In the xy-plane, a circle with radius 5 has center $(-8, 6)$. Which of the following is an equation of the circle?

a. $(x-8)^2+(y+6)^2=5$
b. $(x-8)^2+(y+6)^2=25$
c. $(x+8)^2+(y-6)^2=5$
d. $(x+8)^2+(y-6)^2=25$

Equation of the circle is given by

$$(x-a)^2+(y-b)^2=r^2$$

for a circle centered at (a,b) of radius r

(2) **2266984b** MULTIPLE CHOICE One answer only

$$x^2 + 20x + y^2 + 16y = -20$$

The equation above defines a circle in the xy-plane. What are the coordinates of the center of the circle?

a. $(20, 16)$
b. $(10, 8)$
c. $(-20, -16)$
d. $(-10, -8)$

Idea: bring $x^2+20x+y^2+16y = -20$ to $(x-\text{something})^2 + (y-\text{otherthing})^2 = r^2$

method: completing the square, $(x+sth)^2 = x^2 + 2\cdot sth\cdot x + sth^2$

$(y+sth)^2 = x^2 + 2\cdot sth\cdot x + sth^2$

x^2+20x+ ___ missing: sth^2 → compare... $20x = 2\cdot sth\cdot x \Rightarrow sth = 10$

$x^2 + 2\cdot sth\cdot x + sth^2$

then we add $0 = 10^2 - 10^2$!

$x^2+20x = \underbrace{x^2+20x+10^2}_{=(x-10)^2} \underbrace{-10^2}_{=0}$

$= (x+10)^2 - 10^2$

Similarly...

$y^2+16y = \underbrace{y^2+16y+8^2}_{(y+8)^2} \underbrace{-8^2}_{=0}$

$= (y+8)^2 - 8^2$

Altogether, $x^2+20x+y^2+16x = (x-10)^2 - 10^2 + (y+8)^2 - 8^2 = -20$

$\Rightarrow (x-10)^2 + (y+8)^2 = 8^2 + 10^2 - 20$