11th November

Completed Exercises from the lecture on

< Systems of Equations in Two variables >

1. Hard , Page 2 ;

Bonus, Page 3 ;

Can be found below.

Hard

(1) **fc3d783a** SHORT ANSWER Case-Insensitive

In the xy-plane, a line with equation $2y = 4.5$ intersects a parabola at exactly one point. If the parabola has equation $y = -4x^2 + bx$, where b is a positive constant, what is the value of b ?

$b = 6$

$2y = 4.5 \Rightarrow y = \frac{4.5}{2} = 2.25 = y$

$2.25 = y = -4x^2 + bx$

$2.25 = -4x^2 + bx \quad -2.25$

$\exists \quad 0 = -4x^2 + bx - 2.25 \quad \begin{cases} a = -4 \\ b = b \\ c = -2.25 \end{cases}$

$x_0 = \frac{-b \pm \sqrt{b^2 - 4(-4)(-2.25)}}{-4 \cdot 2}$

$= \frac{-b \pm \sqrt{b^2 - 36}}{-8}$

$\exists \Rightarrow b^2 - 36 = 0$

Bonus Problem

In the xy-plane, the graph of

$$2x^2 - 6x + 2y^2 + 2y = 45$$

is a circle. What is the radius?

$(x-a)^2 + (y-b)^2 = r^2$ graphs a circle centered at (a,b) with radius r.

$$x^2 - 3x + y^2 + y = \frac{45}{2}$$

$$(x - \tfrac{3}{2})^2 - (\tfrac{3}{2})^2 + (y + \tfrac{1}{2})^2 - \tfrac{1}{4} = \frac{45}{2} \qquad + \tfrac{1}{4} + (\tfrac{3}{2})^2$$

$$(x - \tfrac{3}{2})^2 + (y + \tfrac{1}{2})^2 = \frac{45}{2} + (\tfrac{3}{2})^2 + \frac{1}{4} = r^2$$

$$\Rightarrow \sqrt{\frac{45}{2} + (\tfrac{3}{2})^2 + \frac{1}{4}} = 5$$

$$x^2 - 3x = \overbrace{x^2 - 2 \cdot \tfrac{3}{2} x + (\tfrac{3}{2})^2}^{=(x-\frac{3}{2})^2} - \underbrace{(\tfrac{3}{2})^2}_{=0}$$

$$= (x - \tfrac{3}{2})^2 - (\tfrac{3}{2})^2$$

$(x-a)^2 = x^2 - 2 \cdot a \cdot x + a^2$, $a = 1.5$

$(y-b)^2 = y^2 - 2 \cdot b \cdot y + \underline{b^2}$, $-2b = 1 \Rightarrow b = -\frac{1}{2}$

$$= y^2 + 1y$$

$$= y^2 - -\tfrac{1}{2} \cdot 2 \cdot y$$

$$= \overbrace{y^2 - -\tfrac{1}{2} \cdot 2 \cdot y + \tfrac{1}{4}}^{(y--\frac{1}{2})^2} \underbrace{- \tfrac{1}{4}}_{=0}$$

$$= (y - -\tfrac{1}{2})^2 - \tfrac{1}{4}$$