2. Integration ...

. 1 By anti-derivative: the derivative of what gives the integrand?

$\boxed{\int x^2\,dx = \frac{x^3}{3}+c}$. We know $\frac{d}{dx}x^2 = 2x^1$, $\frac{d}{dx}x^5 = 5x^4$

but $\frac{d}{dx}\frac{x^3}{3}+75 = x^2$ then $\frac{d}{dx}\frac{x^3}{3} = \frac{3}{3}x^2 \Rightarrow \frac{d}{dx}\frac{x^3}{3} = x^2$

Another Example: $\int \cos(x)\,dx = \sin(x)+C$

. $\frac{d}{dx}\sin(x)+c = \cos(x)$

$$\boxed{\begin{aligned} &e^\pi \int 1\,dx = e^\pi x + C \\ &\int 1\,dx = x + c \\ &\frac{d}{dx}x = 1 \end{aligned}}$$

one more... $\int e^{3x}\,dx = \frac{1}{3}e^{3x} + C$

$\overbrace{3e^{3x}}$

$e^{3x} = \frac{d}{dx}\frac{e^{3x}}{3} = \cancel{\frac{1}{3}}\cdot\left(\frac{d}{dx}e^{3x}\right)$

Strategy: this works with pure functions, e.g. x^5, $\sin(3x)$, ...

Non-example: $\int \frac{2x}{\sqrt{x^2-4}}\cdot dx$ brings us to...

. 2 By Substitution: u = inner function + vibes

$I = \int \frac{2x}{\underbrace{\sqrt{x^2-4}}_{=u}}\cdot dx$, $u = x^2-4 \Rightarrow du = 2x\,dx$ $\begin{cases} 1.\ \text{replace } dx \\ 2.\ \text{replace all } x \end{cases}$

$\Rightarrow dx = \frac{du}{2x}$

$I = \int \frac{\cancel{2x}}{\sqrt{u}}\cdot\frac{du}{\cancel{2x}} = \int \frac{du}{\sqrt{u}} = \int u^{-\frac{1}{2}}\,du = 2u^{\frac{1}{2}} = \boxed{2\cdot(x^2-4)^{\frac{1}{2}}+C}$

$\boxed{\int x^a\,dx = \frac{x^{a+1}}{a+1}+c}$. $2\frac{d}{dx}u^{\frac{1}{2}} = \cancel{2}\left(\cancel{\frac{1}{2}}\,u^{-\frac{1}{2}}\right) = u^{-\frac{1}{2}}$

Strategy: this works when you have (functions × their derivatives)

Another example: $\int x^3 e^{x^4}\,dx$, $u = x^4 \Rightarrow \frac{du}{4x^3} = \frac{4x^3\,dx}{4x^3}$

$\Rightarrow dx = \frac{du}{4x^3}$

$\int \cancel{x^3} e^{x^4}\frac{du}{4\cancel{x^3}}$

$\frac{1}{4}\int e^u\,du = \frac{1}{4}e^u + C$

$= \frac{e^{x^4}}{4} + C$

one more....: $\int \tan x\,dx = ?$

Non-example: $\int e^x \cos x \, dx$ brings us to...

.3 by Parts: reverse product rule.

$$\boxed{\int u \, dv = uv - \int v \, du} \Rightarrow uv = \int u \, dv + \int v \, du$$

$$\frac{d}{dx} u \cdot v = u \frac{dv}{dx} + v \frac{du}{dx}$$

Back to $I = \int e^x \cos x \, dx$ ... $\boxed{\int u \, dv = uv - \int v \, du}$

$u = \cos x \Rightarrow du = -\sin x \, dx$

$dv = e^x \Rightarrow v = e^x$

$$\int e^x \cos x \, dx = \cos(x) \cdot e^x + \underbrace{\int e^x \sin x \, dx}_{= I_1}$$

$\boxed{\begin{aligned} u &= \sin x \Rightarrow du = \cos x \, dx \\ dv &= e^x \Rightarrow v = e^x \end{aligned}}$

$I_1 = \sin(x) \cdot e^x - \int e^x \cdot \cos x \, dx$

$I_1 = e^x \sin(x) - I$

$$I = \cos(x) \cdot e^x + \underbrace{\boxed{e^x \sin(x) - I}}_{= I_1} \Rightarrow \boxed{I = e^x \cdot \frac{\cos(x) + \sin(x)}{2} + C}$$

Strategy: this works when

- everything fails.
- different-type functions
- Periodic functions: $\sin x$, $\cos x$, e^x

One more! $I = \int \ln(x) \, dx = ?$ $\boxed{\int u \, dv = uv - \int v \, du}$

$u = \ln(x) \Rightarrow du = \frac{1}{x} \cdot dx$

$dv = 1 \cdot dx \Rightarrow v = x$

	D	I
+	$\ln(x)$	1
-	$\frac{1}{x}$	x

$I = \ln(x) \cdot x - \int \cancel{x} \cdot \cancel{\frac{1}{x}} \, dx$

$= \ln(x) \cdot x - \int 1 \, dx$

$\boxed{= \ln(x) \cdot x - x + C}$

Bonus: $\int e^x \cdot x^5 \, dx = e^x (x^5 - 5x^4 + 20x^3 - 60x^2 + 120x)$

$- 120 \int e^x \, dx$

$\boxed{= e^x (x^5 - 5x^4 + 20x^3 - 60x^2 + 120x - 120) + C}$

	D	I
+	x^5	e^x
-	$5x^4$	e^x
+	$20x^3$	e^x
-	$60x^2$	e^x
+	$120x$	e^x
-	120	e^x
+	0	e^x

$$\cosh(x) \underset{\frac{d}{dx}}{\overset{d/dx}{\rightleftarrows}} \sinh(x)$$