1. Calculate $\frac{d}{dt}[a^t]$ where $a>0$ is a constant.

$y = a^t \Rightarrow \frac{d}{dt}\left[\ \ln(y) = \ln(a^t) = t\ln(a)\ \right]$

$\boxed{\ln(x^t) = t\ln(x)}$

$\boxed{[\ln(x)]' = \frac{1}{x}}$

$\Rightarrow \frac{d}{dt}\ln(y) = \frac{1}{y}\cdot\frac{dy}{dt} = \frac{d}{dt}t\ln(a) = \ln(a)$

$\frac{1}{y}\cdot\frac{dy}{dt} = \ln(a) \Rightarrow \boxed{\frac{dy}{dt} = \ln(a)\cdot y = \ln(a)\cdot a^t}$ (1)

*) $y = a^t \Rightarrow \boxed{\frac{dy}{dt} = \frac{d}{dt}a^t}$ (2)

(1), (2) $\Rightarrow \boxed{\frac{d}{dt}a^t = \ln(a)\cdot a^t}$

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2. Calculate $\frac{d}{dt}[A\cdot\cos(wt+e)]$ where A, w, e are constants.

$\boxed{\begin{array}{l} 1.\ [f\cdot g]' = f'\cdot g + g'\cdot f \\ 2.\ [\cos(x)]' = -\sin x \\ 3.\ [f\circ g]' = (f'\circ g)\cdot g' \end{array}}$

$= A[\cos(wt+e)]' + \cancel{\underbrace{[A]'}_{=0}\cdot\cos(wt+e)}$

$\cdot\ [\cos(wt+e)]' = [\cos(t)\circ(wt+e)]'$

$= \left([\cos(t)]'\circ(wt+e)\right)\cdot(wt+e)'$

$\cdot\ (wt+e)' = w(t)' = w$

$= -\left(\sin(t)\circ(wt+e)\right)\cdot w$

$= -\sin(wt+e)\cdot w$

$\Rightarrow \boxed{\frac{d}{dt}[A\cdot\cos(wt+e)] = -Aw\sin(wt+e).}$

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3. If $\cosh(x) = \frac{e^x+e^{-x}}{2}$, $\sinh(x) = \frac{e^x-e^{-x}}{2}$, find $[\cosh(x)]', [\sinh(x)]'$.

$\boxed{\begin{array}{l} 1.\ (e^x)' = e^x \\ 2.\ (f\circ g)' = (f'\circ g)\cdot g' \end{array}}$

$(\cosh(x))' = \frac{1}{2}(e^x+e^{-x})' = \frac{1}{2}[(e^x)' + (e^{-x})']$

$\sinh(x) \quad = \frac{1}{2}[e^x - e^{-x}]$

$(\sinh(x))' = \frac{1}{2}(e^x - e^{-x})' = \frac{1}{2}[(e^x)' - (e^{-x})']$

$\cosh(x) \quad = \frac{1}{2}[e^x + e^{-x}]$

$\boxed{\sinh(x) \xleftarrow{\frac{d}{dx}} \cosh(x), \quad \sinh(x) \xrightarrow{\frac{d}{dx}} \cosh(x)}$

$\cdot\ (e^x)' = e^x$

$\cdot\ (e^{-x})' = (e^x\circ(-x))' \overset{2}{=} (e^x\circ(-x))\cdot(-x)'$

$= -e^{-x}$

4. Calculate $\frac{d}{dx}[\ln(a^x+a^{-x})]$ where $a>0$ is a constant.

$$\boxed{\begin{array}{l} 1.\ (f\circ g)' = (f'\circ g)\cdot g' \\ 2.\ (f+g)' = f'+g' \\ 3.\ (a^x)' = a^x \ln(a) \\ 4.\ (\ln(x))' = \frac{1}{x} \end{array}}$$

$$\frac{d}{dx}\left(\ln(a^x+a^{-x}) = \ln(x)\circ(a^x+a^{-x})\right)$$

$$\overset{(1)}{=} \left(\frac{1}{x}\circ(a^x+a^{-x})\right)\cdot(a^x+a^{-x})'$$

$$\boxed{= \frac{1}{a^x+a^{-x}}\cdot(a^x\cdot\ln(a) - a^{-x})}$$

$$\cdot\ (a^x+a^{-x})' \overset{(2)}{=} (a^x)' + (a^{-x})'$$

$$\overset{(3)}{=} a^x\cdot\ln(a) - a^{-x}$$

5. Calculate $\frac{d^3}{dx^3}[x^4 e^x] = \left(x^4\cdot(e^x)' + e^x\cdot(x^4)'\right)''$

$$\boxed{\begin{array}{l} 1.\ (f\cdot g)' = f'g + g'f \\ 2.\ (x^n)' = n\cdot x^{n-1} \\ 3.\ (e^x)' = e^x \\ 4.\ (f+g)' = f'+g' \end{array}}$$

$$= \left(x^4\cdot e^x + e^x\cdot 4x^3\right)''$$

$$= \left(e^x(x^4+4x^3)\right)''$$

$$= \left(e^x(x^4+4x^3)' + (e^x)'\cdot(x^4+4x^3)\right)'$$

$$= \left(e^x\left[4x^3+12x^2+(x^4+4x^3)\right]\right)'$$

$$= \left(e^x\left[x^4+8x^3+12x^2\right]\right)' =$$

$$= e^x\left(\left[x^4+8x^3+12x^2\right]' + \left[x^4+8x^3+12x^2\right]\right)$$

$$\boxed{= e^x\left(x^4+12x^3+36x^2+24x\right)}$$

6. Calculate $\frac{d}{dx}[x^x]$, let $x^x = y \Rightarrow \frac{d}{dx}x^x = \frac{d}{dx}y = \frac{dy}{dx}$

$$\boxed{\begin{array}{l} 1.\ (\ln(x))' = \frac{1}{x} \\ 2.\ (f\cdot g)' = f'\cdot g + g'\cdot f \\ 3.\ (f\circ g)' = (f'\circ g)\cdot g' \\ 4.\ \ln(b^n) = n\cdot\ln(b) \end{array}}$$

$$\ln(x^x) = \ln(y)$$

$$x\cdot\ln(x) = \ln(y) \qquad \left(\frac{d}{dx}\right)$$

$$\boxed{\begin{array}{l} (x^{x^x})' \\ (x^{e^x})' \end{array}}$$

$$x\cdot(\ln(x))' + (x)'\cdot\ln(x) = \frac{1}{y}\cdot\frac{dy}{dx}$$

$$\cancel{x}\cdot\frac{1}{\cancel{x}} + \ln(x) = \frac{1}{y}\cdot\frac{dy}{dx} \qquad (\cdot y)$$

$$\boxed{(1+\ln(x))\cdot x^x = \frac{dy}{dx} = \frac{d}{dx}x^x}$$

7. Softplus(x) = ln(1+e^x). Find [Softplus(x)]' and its domain of definition.

$[\text{Softplus}(x)]' = [\ln(x) \circ (1+e^x)]' =$

$= \left((\ln(x))' \circ (1+e^x)\right) \cdot (1+e^x)'$

$= \left(\frac{1}{x} \circ (1+e^x)\right) \cdot e^x = \boxed{\frac{e^x}{1+e^x}}$

$= \frac{e^x \cancel{+1} - 1}{\cancel{1+e^x}} = 1 - \frac{1}{\underbrace{1+e^x}_{\geq 1 \ \forall x}}$

1. $(f \circ g)' = (f' \circ g) \cdot g'$
2. $(\ln(x))' = \frac{1}{x}$
3. $(e^x)' = e^x$

$\boxed{9.\ D_{(\text{Softplus})'} = \mathbb{R}}$

$\sum n \cdot x^{n-1}$

$= 1 \cdot x^0 + 2x^1 + 3x^2$

$\sum n \cdot x^n$

$0 \cdot x^0 + 1 \cdot x^1$

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8. $\sum_{n=0}^{\infty} n \cdot x^n$

hint: $\sum_{n=0}^{\infty} x^n = x^0 + x^1 + x^2 + \dots = \frac{1}{1-x}$

$(x^n)' = n \cdot x^{n-1}$

a) $\left(\frac{1}{1-x}\right)^2$

b) $-\frac{x}{1-x^2}$

c) $\boxed{\frac{x}{(1-x)^2}}$

d) $\frac{x}{1-x}$

$\left(\sum_{n=0}^{\infty} x^n\right)' = \left(\frac{1}{1-x} = \frac{1}{x} \circ (1-x)\right)'$

$x \cdot \sum_{n=0}^{\infty} n \cdot x^{n-1} = \left(\left(\frac{1}{x}\right)' \circ (1-x)\right) \cdot (1-x)' \cdot x$

$= \left(-x^{-2} \circ (1-x)\right) \cdot \cancel{-1} \cdot x$

$= x \cdot (1-x)^{-2} = \frac{x}{(1-x)^2}$

$\sum_{n=0}^{\infty} n \cdot x^n =$

1. $(f \circ g)' = (f' \circ g) \cdot g'$
2. $(x^n)' = n \cdot x^{n-1}$
3. $(f+g)' = f' + g'$

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9. $\frac{d}{dx} e^{3x} \cdot \cos 4x = y_1$

$= (e^{3x})' \cos 4x + e^{3x} (\cos 4x)'$

• $(e^{3x} = e^x \circ 3x)' =$

$= (e^x \circ (3x)) \cdot (3x)'$

$= 3e^{3x}$

• $(\cos 4x = \cos x \circ (4x))'$

$= ((\cos x)' \circ (4x)) \cdot (4x)'$

$= 4 \cdot ((-\sin x) \circ (4x))$

$= -4 \cdot \sin 4x$

$\frac{d}{dx} e^{3x} \cdot \sin 4x = y_2$

$= (e^{3x})' \sin 4x + e^{3x} (\sin 4x)'$

• $(\sin 4x = \sin x \circ 4x)'$

$= ((\sin x)' \circ (4x)) \cdot (4x)'$

$= 4(\cos x \circ 4x)$

$= 4 \cos 4x$

$$\boxed{\begin{aligned} y_1 &= e^{3x}(3\cos 4x - 4\sin 4x) \\ y_2 &= e^{3x}(3\sin 4x + 4\cos 4x) \end{aligned}}$$

1. $(f \cdot g)' = f' \cdot g + g' \cdot f$
2. $(f \circ g)' = (f' \circ g) \cdot g'$
3. $(\cos x)' = -\sin x$
4. $(\sin x)' = \cos x$
5. $(e^x)' = e^x$

10. A fly flies along $y = x^3$ such that $x(t) = 2t + 1$. What is its velocity y-component at $t = 1$?

" $x \sim y$ " " $x \sim t$ "

" $y \sim t$ "

" vel = (disp)' "

$$V_y(t) = \frac{d}{dt} y(t) = \frac{dy}{dt}$$

$$y = x^3 \Rightarrow \frac{dy}{dt} = 3x^2 \frac{dx}{dt} = 3 \cdot 2 \cdot x^2 = 6x^2$$

$$x(t) = 2t + 1 \Rightarrow \frac{d}{dt} x(t) = 2 = \frac{dx}{dt}$$

At time $t = 1$, $x(t=1) = 3$

$$\boxed{\Rightarrow \frac{dy}{dt}(t=1) = 6(3)^2 = 54 \text{ units/second.}}$$